Improving SGP4 Orbit Determination with New State Estimation Algorithm

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ABSTRACT

The Simplified General Perturbations 4 Model (SGP4) is a well-known tool for performing satellite orbit determination. However, uncertainties and inaccuracies in the initial state inputs (required by SGP4) degrade the performance of the propagator. We present a new state estimation algorithm that allows for independent computation of these initial inputs using Unscented Kalman Filtering and GPS data from a satellite. The algorithm is tested on real flight data and demonstrates a notable performance improvement over the standard method of orbit determination using SGP4.

CONTEXT

SGP4

The SGP4 algorithm was developed in 1970 by Cranford[1] and makes use of the Brouwer[2] gravity model. It is still used today to quickly calculate ephemerides for space objects. Unlike other orbit propagators which rely on numerical integration methods, SGP4 is an analytical propagator which means it is fast but less accurate.

$$X_t = \begin{Bmatrix} i \\ \Omega \\ e \\ \omega \\ M \\ n \\ B^* \end{Bmatrix}$$

SGP4 requires specific initial conditions in order to propagate the orbit. These conditions take the form of a special orbital state vector (commonly referred to as a TLE) which contains 6 mean Keplerian elements (i, Ω, e, ω, M, n) and an atmospheric drag term (B*). A TLE is used to uniquely describe any Earth orbit at any epoch.

TLEs are available in a public catalog for thousands of space objects. Updated calculations for these public TLEs are distributed on a daily basis by fitting the SGP4 model to observations of the current objects in orbit collected by numerous ground stations.

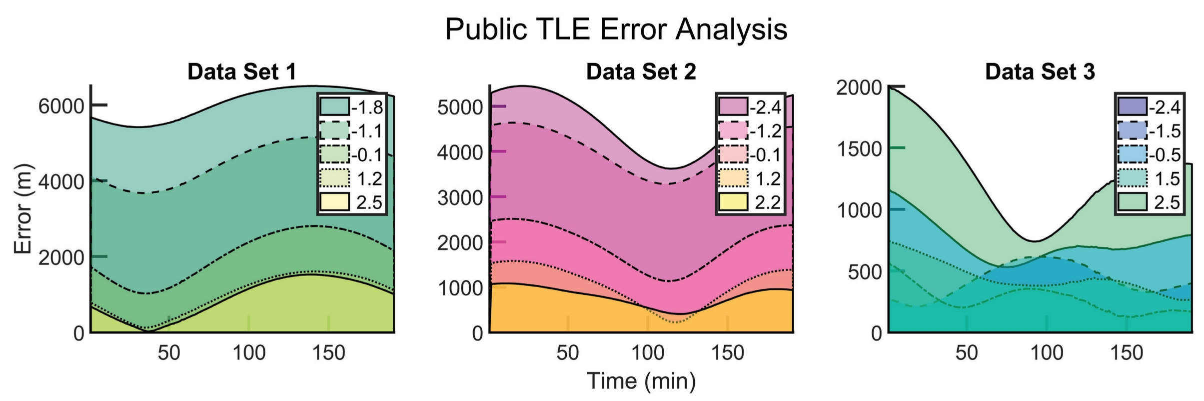


Figure 1. illustrates the accuracy of public TLEs by comparing their associated SGP4 propagation to real flight data. In each test, 5 TLEs were selected that were published within 2.5 days of the data set. Older TLEs tend to be less accurate and newer TLEs tend to be more accurate (though realistically, the "future" TLEs would not be available at the time the data set was recorded). There is a wide range of accuracy both between TLEs with different epochs and TLEs for different data sets spanning a few hundred meters to many kilometers.

Literature Review

Over the years, attempts have been made to circumvent the standard process for obtaining a TLE to make use of independently gathered measurements.

"Fitting Methods" combine multiple data points into a TLE by reverse engineering the standard fitting process directly

"Single Point Methods" aim to convert individual measurements directly into a TLE

Single Point Methods are needed when the data is intermittent or when the data is collected over a short time frame. In such scenarios, fitting methods have been shown to perform poorly.

Problem Statement

Space vehicle position information from GPS (for example) is an untapped source of high-quality data that could be used to calculate TLEs frequently and accurately. Researching and refining the process to convert this data into SGP4 compatible TLEs could provide a trustworthy method to produce TLEs when the public version is unavailable. In addition, this work could help to improve orbital predictions made using the model.

[1] Lane, M. H., and F. R. Hoots. 1979. "Spacetrack Report #2: General Perturbations Theories Derived from the 1965 Lane Drag Theory." Aerospace Defense Command, Peterson AFB, CO
[2] Brouwer, D. 1959. Solution of the Problem of Artificial Satellite Theory without Drag. Astronomical Journal, Vol. 64, No. 1274, pp. 378–397.

ALGORITHM

SPM

The Single Point Method (SPM) uses a recursive root-finding algorithm to find a TLE that satisfies the following equation.

$$SGP4(TLE, t_0) - \vec{r_{t_0}}, \vec{v_{t_0}} = 0$$

SPM relies on calculating 6 osculating elements (O) that describe the Keplerian orbit tangential to a given Cartesian state measurement. These elements have similar (but not identical) values to the 6 corresponding mean elements found in a TLE. Because of this, it is possible to propagate O using SGP4 (although the result will not be very accurate). Calculating O using the known function OSC is easy, unlike calculating a TLE from the *unknown* inverse of SGP4.

1. Convert measurement into osculating elements: $OSC(\vec{r_{t_0}}, \vec{v_{t_0}}) = O_{t_0}$
2. Let initial estimate of TLE equal osculating elements: $TLE_0 = O_{t_0}$
3. Convert from Keplerian to Cartesian state using SGP4: $SGP4(TLE_i, t_0) = \vec{r_i'}, \vec{v_i'}$
4. Convert Cartesian state back into osculating elements: $OSC(\vec{r_i'}, \vec{v_i'}) = O_i'$
5. Isolate perturbations added during SGP4 propagation: $\Delta O_i = O_{t_0} - O_i'$
6. Update TLE by correcting for perturbations: $TLE_{i+1} = TLE_i + \Delta O_i$ (loops back to step 3)

In SGP4, perturbation terms are added to the mean elements to account for non-ideal Keplerian orbits. Therefore, SGP4 only works with TLEs in which corresponding perturbations have been removed. Osculating elements already include perturbations and so each iteration of the loop distorts the osculating elements by adding erroneous perturbations.

As these steps are repeated, the TLE converges such that the SGP4 output position and velocity matches the input measurement position and velocity. The loop is terminated when the two positions and velocities agree to a certain tolerance.

UKF

The underlying assumption of the SPM is that the input measurement is perfectly accurate. In practice, noisy measurements will translate to noisy mean elements. Combining SPM with an Unscented Kalman Filter (UKF) can help to reduce this noise and improve the TLE estimate. SPM iterates on a single measurement while UKF iterates over multiple measurements.

The UKF follows two general steps: Prediction and Correction

Prediction

- "Sigma points" are generated that capture the mean and covariance of the current state: $\chi_{i-1} = f(x_{i-1}, P_{i-1})$
- The points are propagated forward using SGP4 and SPM and converted back to mean elements: $\chi_i = SPM[SGP4(\chi_{i-1}, t_i)]$
- The points are combined into a predicted state, predicted measurement, and predicted errors: $\chi_i \rightarrow x_i', z_i', P_{x_i}', P_{zz_i}', P_{xz_i}'$

Correction

- Residual is calculated from predicted measurement and real measurement: $y_i = z_i - z_i'$
- Kalman gain is calculated from predicted errors: $K = P_{xz_i}/P_{zz_i}$
- State and state error are updated using residual and Kalman gain: $x_i = x_i' + KR$, $P_i = P_{x_i} - (KP_{zz_i}K^T)$ (loops back to Prediction)

RESULTS

Data Set	SV2	SV3
Duration (min)	130	95
Measurements (#)	261	286
Sample Rate (Hz)	1/30	1/20
Previous TLE (hr)	−11.5	−2.8

The algorithms were applied to two data sets containing on-orbit GPS position and velocity information from two space vehicles (SV2 & SV3). The standard deviation for both datasets is approximately 6m along each axis of position and 0.7m/s along each axis of velocity.

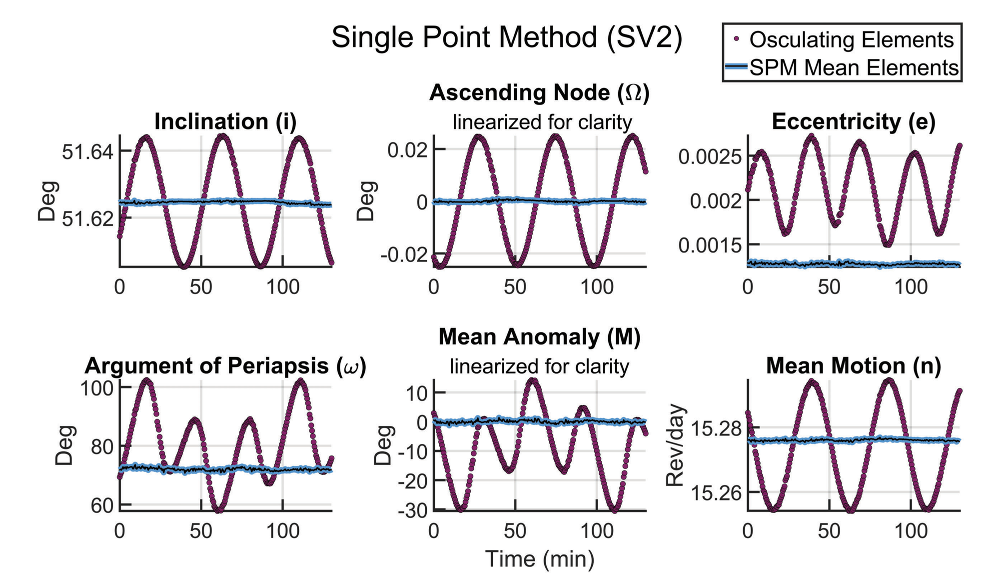


Figure 2. shows the SPM algorithm applied to the SV2 data set. For each GPS measurement, the raw osculating elements as well as the converged mean elements are shown. On average, SPM took 50ms per data point to run and took 15 iterations per data point to reach the tolerance threshold of 1cm and 1cm/s for position and velocity agreement.

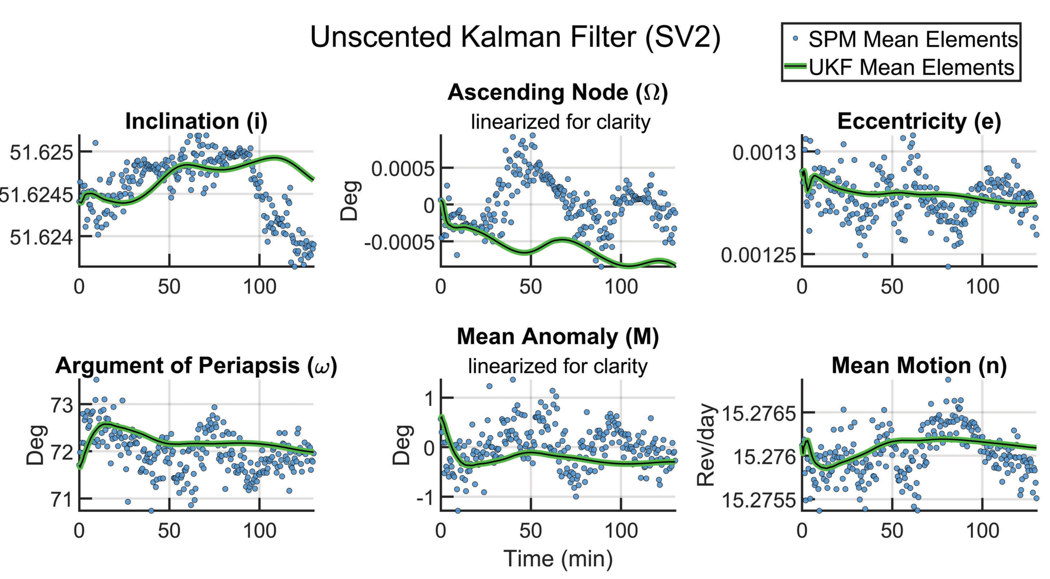


Figure 3. shows how the UKF can smooth out the noise in the mean elements generated using SPM. The filter takes approximately 40 data points to converge, after which it continues to remain sensitive to new inputs. The UKF processes incoming measurements at a speed of approximately 800ms per data point (each iteration of UKF invokes SPM for 15 sigma points).

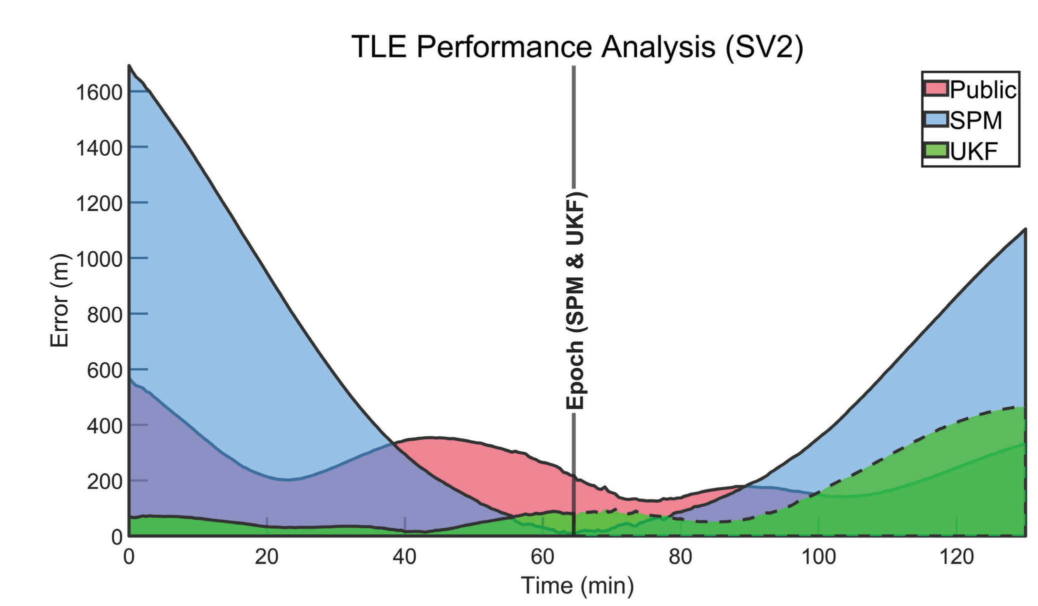


Figure 4. shows a raw performance comparison of the different TLEs applied to the SV2 data set. The SPM and UKF TLEs were sampled from the middle of the data set. Each TLE was propagated using SGP4 and the resulting ephemeris was compared with the GPS data to compute the error. The UKF shows clear improvement over SPM and matches the performance of the public TLE.

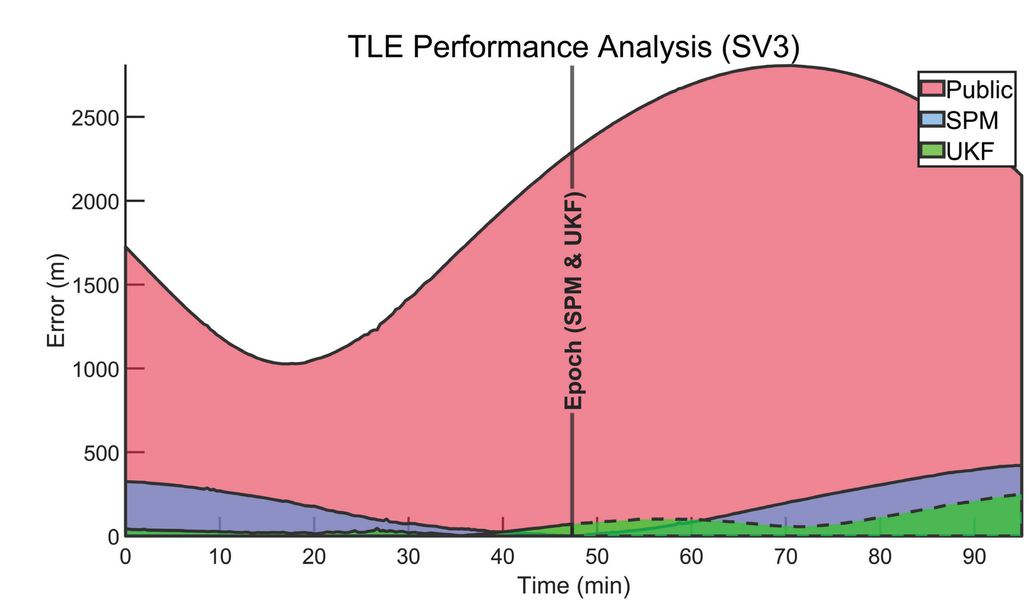
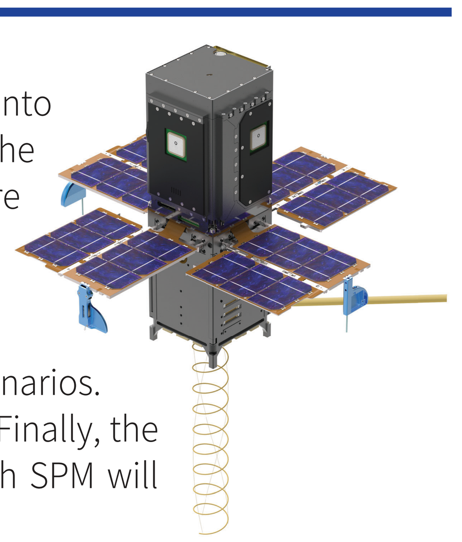


Figure 5. shows the same TLE performance comparison for the SV3 data set. In this case, the public TLE for SV3 performs worse than the public TLE for SV2 despite being only a few hours old vs half a day old. Consequentially, both the SPM and the UKF show a significant performance improvement over the public TLE with the UKF having 95 percent better accuracy overall.

SUMMARY



The new state estimation algorithm combines a "Single Point Method" for converting GPS measurements into TLEs, with an Unscented Kalman Filter. While SPM can provide perfect instantaneous performance, the method does not produce reliable predictions into the future. UKF, on the other hand, can predict future orbits to a higher degree of accuracy and consistency. In one case, the UKF algorithm was 20 times more accurate than the public TLE.

More data set testing is needed to fully evaluate the performance across a wider range of orbital scenarios. Additionally, the algorithm needs to be optimized for speed in order to run on a satellite microprocessor. Finally, the robustness of the algorithm can be improved by developing methods to deal with data points on which SPM will sometimes diverge.

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